RWANDA NATIONAL EXAMINATIONS COUNCIL

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105

NATIONAL EXAMINATION 2001/2002.

SUBJECT: MATHEMATICS II

OPTION: BIOLOGY – CHEMISTRY

DURATION: 3 HOURS

INSTRUCTIONS:

- This paper consists of two sections A and B. Section A contains 14 short compulsory questions, which carry a total of 55 marks. Section B contains 5 questions out of which a candidate has to choose only three. Each question in section B carries 15 marks. All working should be shown on the answer sheet provided.

- Tables and/or calculators may be used.

SECTION A (55 marks)

1. Find the first derivative of the function f defined by:

$$f(x) = \cos x - 3\sqrt[3]{x^2} - x e^{-x}$$ **(2marks)**

2. Consider the second degree equation in x:
$mx^2 - (m-1)x + 2m-2 = 0$, where m is a real number. Find the values of m such that the equation has two distinct solutions less than 2. **(6marks)**

3. Solve the equation: $\sin^2 x - \sin x - 2 = 0$. **(5marks)**

4. Find the matrix $\begin{pmatrix} x & y \\ u & v \end{pmatrix}$ such that $\begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} x & y \\ u & v \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 5 & 4 \end{pmatrix}$

(3marks)

5. A numerical function f(x) is defined by: $f(x) = \frac{1}{4} e^x \ln(e^{2x} - 1)$.
Determine the domain of definition of f and find the coordinates of the points of intersection of the curve with the x-axis. **(4marks)**

6. In how many ways can three people sit in a car with 5 seats in such a way that (a) the car remains stationary. **(1mark)**
(b) one of them drives the car and all the three have driving permits? **(1mark)**

7. Solve the inequation $\log(x-1) + \log(x+2) \leq 1$ **(4marks)**

8. Consider the function f: IR → IR defined by $f(x) = \frac{2x^2}{|2-x| + x}$

(a) Study the continuity of the function at the point x = 2. **(2.5marks)**

(b) Write the equation of the asymptote to the curve of f (x). **(1.5marks)**

9. Given the vectors $\vec{u}(1, -1, 2)$, $\vec{v}(2, 0, -1)$ et $\vec{w}(3-x, 2, -3)$ in a space with a rectangular basis, find the vdue of x such that the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ are coplanar. **(3marks)**

10. Using Cramer's method solve the simultaneous equations:

$$\begin{cases} x + 4y + 6z = 0 \\ x + 2y + 2z = 2 \\ 3x - 2y - z = 5 \end{cases}$$

(5marks)

11. Evaluate $\int_{-2/3}^{2/3} \frac{1}{\sqrt{16-9x^2}} dx$ **(4marks)**

12. Find the modulus and the argument of each of the solutions to the complex equation $z^2 + (6\cos a) z + 9 = 0$, $0 < a < \pi/2$. **(6marks)**

13. Consider the parabola whose equation is $y^2 = 4x$. Write down the equation of the tangents to the parabola at symmetric points with reference to the focus of the parabola. **(4marks)**

14. Consider a game in which a player has chances of gaining or losing the first part. If a player gains a given part, the probability that he gains the next part is 0.6. If he loses a part, the probability that he loses the next part is 0.7. Let G_n represent the event "The player gains the n^{th} part";

P_n represent the event "The player loses the n^{th} part".

(a) Determine the probabilities: $p(G_1)$, $p(G_2/G_1)$ and $p(G_2/P_1)$. Hence find the probability $p(G_2)$.

(b) Find the probability $p(P_2)$. **(3marks** in all**)**

SECTION B.

15. Five Companies that offer Security Services have proposed different salaries to people they want to employ. Let x be the proposed salary and y the number of candidates applying for a job. The table below shows the obtained data:

Company	Salary (RWF)	Number of Candidates
1	44,000	10
2	45,000	13
3	46,000	17
4	47,000	19
5	48,000	21

(i) Find the equation of the regression line of y as a function of x. **(12marks)**

(ii) Find the coefficient of linear correlation between x and y. **(3marks)**

16. Let $(\vec{e}, \vec{u})$ be the basis of a real vector space (IR, V, +) with rectangular coordinates and whose vectors are:

$\vec{x} = a(3\vec{e} - 4\vec{u})$ and $\vec{y} = b(\vec{e} + 3\vec{u})$ where a and b are non zero scalars.

(i) Can $(\vec{x}, \vec{y})$ be a basis for the vector space V over IR? **(7½marks)**

(ii) Suppose it is found to be true that $(\vec{x}, \vec{y})$ can be a basis for the vectors space V, which relation must a and b verify such that the basis is rectangular? **(7½marks)**

17. For all natural whole numbers n, consider a numerical function f_n defined by $f_n(x) = \frac{x^n}{1+x^2}$, where x is a real number.

Let $I_n = \int_0^1 f_n(t)\,dt$.

(i) Evaluate I_1 **(2marks)**

(ii) Evaluate $I_1 + I_3$, hence find the value of I_3. **(3marks)**

(iii) Show that $I_{2p} + I_{2p+2} = \frac{1}{2p+1}$, where p is a natural whole number. **(3marks)**

(iv) Evaluate I_2, I_4 and I_6. **(6marks)**

18. Consider a polynomial with complex coefficients defined by
$f(z) = z^4 - 4(1+i)z^3 + 12iz^2 - 8i(1+i)z - 5$

(a) Find (i) f (1) **(2marks)**

(ii) f (i) **(2marks)**

(b) Solve the complex equation f (z) = 0. **(11marks)**

19. Make a complete analysis (study) of the numerical function f defined by $f(x) = \frac{\ln x}{x}$ and, using a scale of 1 unit to represent 2 cm, calculate the area of the region enclosed by the graph of the function and the lines x = 1 and $x = e\sqrt{e}$. **(11marks + 4marks)**

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